Sirius
International Mathematics Center

046w: 10th Russian-Chinese Conference on Knot Theory and Related Topics

SEPTEMBER 30–OCTOBER 04 | 2024

Sirius International Mathematics Center

10th Russian-Chinese Conference on Knot Theory and Related Topics

September 30 – October 4, 2024

Program and Abstracts

Sirius Federal Territory, 2024

Organizers

Andrei Vesnin	Sobolev Institute of Mathematics of the SB RAS & Tomsk State University
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Vassily Manturov	Moscow Institute of Physics and Technology
Andrey Vesnin	Sobolev Institute of Mathematics of SB RAS & Tomsk State University
Jiajun Wang	Peking University
Zhiqing Yang	Dalian University of Technology
Ying Zhang	Soochow University

Knot theory is an actively developing branch of geometry and topology. Modern knot theory includes the study of knots in thickened surfaces and other three-dimensional manifolds, notoids, and knotted graphs. It is characterized by a combination of methods of three-dimensional topology, algebraic topology, group theory, representation theory, and non-Euclidean geometry.

The purpose of the conference is to present new results and discuss open problems related to current trends in knot theory.

This is the 10th anniversary conference in the ongoing series of China-Russia conferences on knot theory and related topics, which are held alternately in China and Russia.

Web-pages: https://nomc.math.tsu.ru/10CRCKT
https://siriusmathcenter.ru/program_046w
Zoom: https://tinyurl.com/ydcpfkkk (time in UTC+3/GMT+3)

Sirius International Mathematics Center, Sirius Federal Territory
Regional Mathematical Center of Tomsk State University, Tomsk

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Conference Program

SEPTEMBER 30, MONDAY

09^{10} — 09^{20} REGISTRATION
TURIN CONFERENCE HALL, OMEGA SIRIUS HOTEL

09^{20} — 09^{30} OPENING

09^{30} — 10^{20} Akio Kawauchi (online)
Classifying the surface-knot modules

10^{30} — 11^{20} Vassily Manturov
TBA

COFFEE BREAK

11^{50} — 12^{40} Zhiyun Cheng
Intersection graph and writhe polynomial

12^{50} — 13^{40} Sabir Gusein-Zade
Algebraic links and 'real' algebraic links; a possible real version of knots and links

LUNCH BREAK

15^{00} — 15^{25} Tatyana Kozlovskaya
Defects of linear representations

15^{30} — 15^{55} Qing Liu
Growth of stable subgroups in Morse-local-to-global groups

16^{00} — 16^{25} Andrey Egorov
Volume bounds for hyperbolic links in terms of number of twists in the diagram

COFFEE BREAK

OCTOBER 1, TUESDAY

$09^{30} - 10^{20}$ Sang Youl Lee (online)
Invariants for colored links using multi-biquandles

$10^{30} - 11^{20}$ Andrey Vesnin
Spatial graphs and associated links

COFFEE BREAK

$11^{50} - 12^{40}$ Jiajun Wang
Heegaard Floer homology and the fundamental group

$12^{50} - 13^{40}$ Alexander Mednykh
Plans's theorem for knots and Jacobians of graphs

LUNCH BREAK

$15^{00} - 15^{50}$ Wenyuan Yang
Growth tightness of quotients by confined subgroups

$16^{00} - 16^{25}$ Zhe Sun
Intersections of dual $SL3$-webs

COFFEE BREAK

$17^{00} - 17^{25}$ Alexey Miller
Geometric properties of surgery graphs in low-dimensional topology

$17^{30} - 17^{55}$ Yu Pan
Augmentations and exact Lagrangian surfaces

OCTOBER 3, THURSDAY

$09^{30} - 10^{20}$ Ivan Dynnikov
Commutation phenomenon in the rectangular diagram formalism

$10^{30} - 11^{20}$ Jiming Ma
Figure-eight knot is always over there

COFFEE BREAK

$11^{50} - 12^{40}$ Andrei Malyutin
Some knot theory conjectures related to the crossing number additivity conjecture

$12^{50} - 13^{40}$ Nikolay Abrosimov
Euclidean volume of a cone manifold over any hyperbolic knot is an algebraic number

LUNCH BREAK

$15^{00} - 15^{50}$ Sergei Melikhov
Is every knot isotopic to the unknot?

$16^{00} - 16^{25}$ Artem Belov
Measuring Chern-Simons level k by braiding $SU(2)_k$ anyons

COFFEE BREAK

$17^{00} - 17^{50}$ Louis H. Kauffman (online)
Graph coloring, Penrose formulas and multi-virtual knot theory

OCTOBER 4, FRIDAY

$09^{30} - 10^{20}$ Rinat Kashaev (online)
Knot polynomials from braided Hopf algebras with automorphisms

$10^{30} - 11^{20}$ Valeriy Bardakov
Handlebody-links and spatial graphs

COFFEE BREAK

$11^{50} - 12^{40}$ Dmitry Talalaev
Reflection equation algebras, quantum Toda system and quasideterminants

$12^{50} - 13^{40}$ Igor Nikonov
Partial tribrackets of knots in thickened surfaces

LUNCH BREAK

$15^{00} - 15^{50}$ Xuezhi Zhao
Surfaces in Seifert manifolds

$16^{00} - 16^{25}$ Bao Vuong
On links in Poincare homology sphere

COFFEE BREAK

$17^{00} - 17^{25}$ Maxim Ivanov
Virtual knot groups and circular orderability

$17^{30} - 17^{55}$ Ilya Alekseev
Positive braids and the HOMFLY-PT polynomial

Abstracts

Euclidean volume of a cone manifold over any hyperbolic knot is an algebraic number

03.10
12:50–13:40

Nikolay Abrosimov

Sobolev Institute of Mathematics & Tomsk State University

The talk is based on our joint work with Alexander Kolpakov (Université de Neuchâtel, Switzerland) and Alexander Mednykh (Sobolev Institute of Mathematics, Novosibirsk). A hyperbolic structure on a three-dimensional cone manifold with a knot as a singular set can usually be deformed into a limit Euclidean structure. In our paper [AKM24] we show that the corresponding normalized Euclidean volume of the manifold is always an algebraic number, i.e., a root of some polynomial with integer coefficients. This result is a generalization (for cone manifolds) of the well-known Sabitov theorem on the volumes of Euclidean polyhedra, which gave an answer to the bellows problem. The fact we established stands out against the background of hyperbolic volumes, the number-theoretic nature of which is usually quite complex. In addition to this theorem, we propose an algorithm that allows one to explicitly calculate the minimal polynomial for a normalized Euclidean volume.

Example.

Cone manifold over a knot 5_2 has a normalized Euclidean volume

$$\frac{1}{6\sqrt{-6+68\sqrt{2}+4\sqrt{983+946\sqrt{2}}}} = 0,009909630999945638...$$

Its minimal polynomial is

$$1 + 864x^2 - 64457856x^4 - 412091172864x^6 - 785065068490752x^8.$$

[AKM24] N. Abrosimov, A. Kolpakov, and A. Mednykh, *Euclidean volumes of hyperbolic knots*, Proc. Amer. Math. Soc. **152** (2024), pp. 869–881, DOI: https://doi.org/10.1090/proc/16353.

04.10
17:30–17:55

Positive braids and the HOMFLY-PT polynomial

Ilya Alekseev

Euler International Mathematical Institute

The talk is devoted to the structure of the set of closed positive braids that satisfy the equality in the Morton–Franks–Williams inequality. All such closed braids realize the minimal crossing number, the minimal number of strands, and the maximal self-linking number. We compare the above set with known classes of braids satisfying similar properties. Besides, we refine the result of Gonzalez-Meneses and Manchon that provides a combinatorial characterization of positive braids satisfying the equality in the Morton–Franks–Williams inequality.

04.10
10:30–11:20

Handlebody-links and spatial graphs

Valeriy Bardakov

Sobolev Institute of Mathematics & Tomsk State University

The handlebody-knot theory is a generalization of the classical knot theory and is a 'quotient' of the theory of spatial 3-valent graphs by so called IH-move. We discuss some known invariants for handlebody-knots and spatial graphs. In particular, we recall a G-family of quandles that is an algebraic construction which was proposed by A. Ishii, M. Iwakiri, Y. Jang, K. Oshiro in 2013. The axioms of these algebraic systems were motivated by handlebody-knot theory and give a possibility to define some invariants. In the present work we investigate possible constructions which generalise G-family of quandles and other similar constructions. We provide the necessary conditions under which the resulting algebraic system gives a colouring invariant of handlebody-knots.

This is joint work with Denis Fedoseev.

Measuring Chern-Simons level k by braiding $SU(2)_k$ anyons

03.10
16:00–16:25

Artem Belov

Moscow Institute of Physics and Technology

Recently there was published quite a large number of papers devoted to the derivation of quantum gates for topological quantum computations in Chern-Simons theory with predetermined parameters. For such calculations it is supposed to use world lines of hypothetical particles – anyons. However, any inquisitive mind can have a question: here the experimenter for the first time has received a material with anyons. How can one determine to which theory the resulting anyons belong? What properties they should have for quantum calculations? What difficulties might the experimenter encounter? What sequence of actions should be taken to calculate the parameters of the theory? These are the key questions that are the focus of this report.

We will discuss an algorithm for calculating the parameter k in a material with $SU(2)$ anyons and possible difficulties that an experimenter may encounter during measurements.

Intersection graph and writhe polynomial

30.09
11:50–12:40

Zhiyun Cheng

Beijing Normal University

In this talk, I will explain the relation between the intersection graph of a chord diagram and the writhe polynomial of the corresponding virtual knot.

Commutation phenomenon in the rectangular diagram formalism

03.10
09:30–10:20

Ivan Dynnikov

Steklov Mathematical Institute

In recent years, jointly with Maxim Prasolov and Vladimir Shastin, we developed a method for distinguishing Legendrian and transverse links, and solved the problem of their algorithmic recognition. The proof is based on a combinatorial formalism of rectangular diagrams representing knots and links in the three-sphere. The nice feature of those diagrams is their tight relation to contact topology. Each rectangular diagram (of a link or surface) represents not one but two distinct objects interesting from the contact topology point of view, and the contact' properties of each of these objects are independent' of those of the other. We express this independence by saying that type I moves of rectangular diagrams commute with type II moves. It is what makes rectangular diagrams a powerful tool in studying Legendrian and transverse links. A weak form of this commutation phenomenon was observed in my joint paper with Maxim Prasolov in 2013, where it allowed us to prove Jones' conjecture and to find an algorithm for computing the maximal Thurston–Bennequin number of a knot. In my talk I will explain what the above commutation means and how it helps to establish the mentioned results.

This work was supported by the Russian Science Foundation under grant no. 22-11-00299.

30.09
16:00–16:25

Volume bounds for hyperbolic links in terms of number of twists in the diagram

Andrey Egorov

Novosibirsk State University & Tomsk State University

The appendix to [Lac04] provides an upper bound for the volume of hyperbolic link in terms of the number of twists in its diagram. I will talk about a new upper bound for the volume of hyperbolic links, which improves the bound from [Lac04] if the link diagram has more than eight twists.

[Lac04] M. Lackenby, *The volume of hyperbolic alternating link complements. With an appendix by I. Agol and D. Thurston*, Proceedings of the London Mathematical Society **88** (2004), pp. 204–224.

Algebraic links and 'real' algebraic links; a possible real version of knots and links

30.09
12:50–13:40

Sabir Gusein-Zade

Lomonosov Moscow State University

An algebraic link is the intersection of a germ of a plane analytic curve $(\mathbb{C}, 0) \subset (\mathbb{C}^2, 0)$ (reducible or irreducible) with the sphere S^3_ε of a small radius ε centred at the origin. To an algebraic link one associates an analytic invariant: the so-called Poincaré series. For an irreducible curve germ it is defined in the following way. Let $\varphi : (\mathbb{C}, 0) \to (\mathbb{C}, 0)$ be a parametrization (an uniformization) of the curve $(\mathbb{C}, 0)$. For a germ $f \in \mathcal{O}_{\mathbb{C}^2, 0)}$ of a function in two variables, let $v(f)$ be the degree of the leading term in the power series decomposition $f \circ \varphi(\tau) = a\tau^{v(f)} +$ terms of higher degree. If $f \circ \varphi \equiv 0$, $v(f) := +\infty$ (v is a valuation on the ring $\mathcal{O}_{\mathbb{C}^2, 0)}$.) For $k \in \mathbb{Z}$, let $J(k) = \{f \in \mathcal{O}_{\mathbb{C}^2, 0)} : v(f) \geq k\}$. The Poincaré series is $P_{\mathbb{C}}(t) = \sum_(k = 0)^\infty \dim J(k)/J(k+1)t^k$. It appears that the Poincaré series $P_{\mathbb{C}}(t)$ coincides with the Alexander polynomial of the link (divided by $(1 - t)$ for a knot). (Each of these invariants is cyclotomic, i.e., a (finite) product/ratio of binomials of the form $(1 - t^m)$.) There exists a generalization of the described coincidence of analytic and topological invariants of algebraic links for the HOMFLY polynomial: A. Oblomkov, V. Shende, and D. Maulik.

Assume that the complex plane has a fixed structure as the complexification of the real one. One can consider algebraic links in the three-sphere S^3_ε with this additional structure. In this setting, there are at least two versions of the Poincaré series. (Both of them seem to be cyclotomic. They are cyclotomic in the case of knots.) In general, a possible version of this construction for arbitrary links or knots can be like that. Let us consider the sphere S^3 in the complex plane centred at the origin with the involution of the complex conjugation. A link in it is a 1-dimensional submanifold L such that $L \cap \sigma(L) = \emptyset$. There is a problem to define an analogue of the Alexander polynomial for it and to compare it with the Poincaré series.

04.10
17:00–17:25

Virtual knot groups and circular orderability

Maxim Ivanov

Sobolev Institute of Mathematics & Tomsk State University

A group G is called left-orderable if there is an order on G that is invariant under left multiplication. There is a related notion of a circularly orderable group, which states whether or not elements of a group G can be "arranged in a circle" in a way that the relative position of the elements of G on a circle is preserved by left multiplication. All classical knot groups are left-orderable by the famous theorem of Howie and Short. Virtual knots were introduced by L. Kauffman as a generalization of classical knots. Virtual knot groups do not have this property in general. We will discuss left-orderability and circular orderability of these groups.

04.10
09:30–10:20

Knot polynomials from braided Hopf algebras with automorphisms

Rinat Kashaev

Université de Genève

Given a braided Hopf algebra endowed with an automorphism, one can construct an R-matrix over the underlying vector space of this braided Hopf algebra. In the case of Nichols algebras, this leads to multivariable knot polynomials generalising those related to Borel parts of small quantum groups. In the case of a generic Nichols algebra of rank 1, our construction reproduces the sequences of coloured Jones and ADO polynomials, while in the case a particular Nichols algebra of diagonal type of rank 2, we obtain a sequence of 2-variable knot polynomials, which starts with the Links-Gould polynomial, and whose second polynomial detects the Seifert genus of all knots of up to 15 crossings.

This is a joint work with Stavros Garoufalidis.

Graph coloring, Penrose formulas and multi-virtual knot theory

03.10
17:00–17:50

Louis H. Kauffman

University of Illinois at Chicago

We begin by discussing proper edge (three) colorings of trivalent graphs.

Peter Guthrie Tait reformulated the four color theorem in terms of such graphs in the 1880's. One of the advantages of the edge colorings is that there are non-trivial uncolorable graphs that are not planar, such as the Petersen graph. In the 1960's Roger Penrose published the paper "On Applications of Negative Dimensional Tensors" in which he gave a recursive diagrammatic formula that counts the number of proper edge three colorings of planar trivalent graphs. We will begin by giving our proof of the Penrose formula, and show how it can be generalized to count the number of proper colorings for non-planar trivalent graphs, thus allowing us to extend the reach of this way of thinking beyond the plane.

We then show how n colorings for graphs with perfect matchings can be considered with n greater than 3, and how these ways of thinking lead to new invariants of generalized knots that we call multi-virtual knots and to a theory of multi-virtual braids. The main applications of this talk are both graph theoretic and topological.

Classifying the surface-knot modules

30.09
09:30–10:20

Akio Kawauchi

Osaka Central Advanced Mathematical Institute & Osaka Metropolitan University

The kth module of a surface-knot of a genus g in the 4-sphere is the kth integral homology module of the infinite cyclic covering of the surface-knot complement. The reduced first module is the quotient module of the first module by the finite sub-module defining the torsion linking. It is shown that the reduced first module for every genus g is characterized in terms of properties of a finitely generated module. As a by-product, a concrete example of the fundamental group of a surface-knot of genus g which is not the fundamental group of any surface-knot of genus $g-1$ is given for every $g>0$. The torsion part and the torsion-free part of the second module are determined by the reduced first module and the genus-class on the reduced first module. The third module vanishes. The concept of an exact leaf of a surface-knot is introduced, whose linking is an orthogonal sum of the torsion linking and a hyperbolic linking.

30.09
15:00–15:25

Defects of linear representations

Tatyana Kozlovskaya

Tomsk State University

The Lawrence-Krammer-Bigelow representation is one of the most famous linear representations of the braid group. Lawrence constructed a family of representations of the braid group. Krammer and Bigelow proved that one of these representations is faithful. This leads to a natural question regarding the linearity of the singular braid group. O. Dasbach and B. Gemein constructed a faithful linear representation of the singular 3-strand braid monoid. This representation is an extension of the Burau representation. In my talk I discuss extensions of known representations of the braid group to representation of the singular braid monoid and the singular braid group. In particular we construct an extension of the Lawrence-Krammer-Bigelow representation to the singular braid group. To compare two representations of the same dimensions of a group, we introduce additive and multiplicative defects. We find these defects for extension of the Lawrence-Krammer-Bigelow representation and exterior square of two extensions of the Burau representation.

01.10
09:30–10:20

Invariants for colored links using multi-biquandles

Sang Youl Lee

Pusan National University

An *n-colored link* is a smooth imbedding of n circles in the 3-sphere S^3 such that each circle is colored by one of the numbers $1, \ldots, n$. To date, several invariants for n-colored links have been discovered. For example, multivariable Alexander polynomial, Conway potential function, colored Jones polynomial, and so on. In this talk, I'd like to introduce the notion of multi-biquandle and give some algebraic invariants for n-colored links by using diagrammatic techniques associated with multi-biquandles, and further to construct state-sum invariants for n-colored links by using shadow biquandle 2-cocycles associated with shadow multi-biquandles.

Growth of stable subgroups in Morse-local-to-global groups

30.09
15:30–15:55

Qing Liu
Nankai University

Non-elementary hyperbolic groups grow exponentially faster than their infinite index quasiconvex subgroups. This is a result of Dahmani, Futer, and Wise. For the torsion-free groups or residually finite subgroups, Cordes, Russell, Spriano, and Zalloum investigate the growth of stable subgroups for groups with the Morse local-to-global property. Stable subgroups of finitely generated groups generalize quasi-convex subgroups of hyperbolic groups. We show that the Morse local-to-global groups grow more quickly than their infinite index stable subgroups.

This is joint work with Suzhen Han.

Figure-eight knot is always over there

03.10
10:30–11:20

Jiming Ma
Fudan University

It is well known that the complex hyperbolic triangle group $\Delta(3, 3, 4)$ generated by three complex reflections I_1, I_2, I_3 in $PU(2, 1)$ has a 1-dimensional moduli space. Deforming the representations from the classical R-Fuchsian one to $\Delta(3, 3, 4; \infty)$, that is, when $I_3I_2I_1I_2$ is accidental parabolic, the 3-manifolds at infinity change, from a Seifert 3-manifold to the figure-eight knot complement.

When $I_3I_2I_1I_2$ is loxodromic, there is an open set $\Omega \subset \partial H^2_C = S^3$ associated to $I_3I_2I_1I_2$, which is a subset of the discontinuous region. We show the quotient space $\Omega/\Delta(3, 3, 4)$ is always the figure-eight knot complement in the deformation process. This gives the topological/geometrical explanation that the 3-manifold at infinity of $\Delta(3, 3, 4; \infty)$ is the figure-eight knot complement. In particular, this confirms the conjecture of Falbel-Guilloux-Will. This is joint work with Baohua Xie.

03.10
11:50–12:40

Some knot theory conjectures related to the crossing number additivity conjecture

Andrey Malyutin

St. Petersburg Department of Steklov Mathematical Institute & Steklov Mathematical Institute

We will discuss a collection of problems, conjectures, and results related to the conjecture on the additivity of the crossing number for knots under connected sum. A part of this collection consists of 'local' counterparts of the conjecture, where 'local' means that these counterparts deal with certain properties of crossings and small fragments in (minimal) knot diagrams. Another part concerns statistical characteristics of the (possibly empty) set of knots that do not satisfy the additivity conjecture.

30.09
10:30–11:20

TBA

Vassily Manturov

Moscow Institute of Physics and Technology

01.10
12:50–13:40

Plans's theorem for knots and Jacobians of graphs

Alexander Mednykh

Sobolev Institute of Mathematics

Plans's theorem [Pla53] states that for odd n the first homology group of the n-fold cyclic covering of the three-dimensional sphere branched over a knot is the direct product of two copies of an Abelian group. A similar statement holds for even n, in which case one has to take the quotient of the homology group of the n-fold covering by the reduced homology group of the twofold covering. A modern proof of this theorem can be found in [Gor71] and [Ste96]. The aim of this

section is to establish similar results for the Jacobian groups (critical groups) of circulant graphs. Moreover, it will also be shown that the Jacobian group of a circulant graph on n vertices reduced modulo a fixed finite Abelian group is a periodic function of n. In [MM23] we noticed some parallels between results describing the homology groups of branched cyclic coverings over knots and results in the theory of cyclic coverings over graphs.

We present a correspondence between objects of knot theory and their analogues in graph theory:

- a knot K in the sphere $\mathbb{S}^3$ corresponds to a vertex v of the cone $\hat{G} = \{v\} \star G$;
- the Alexander polynomial of K corresponds to the associated Laurent polynomial of the graph G;
- the complement $\mathbb{S}^3 \backslash K$ corresponds to the graph G;
- a cyclic covering over $\mathbb{S}^3 \backslash K$ corresponds to a cyclic covering over the graph G;
- the cyclic covering M_n of the sphere $\mathbb{S}^3$ branched over K corresponds to the cyclic covering $\hat{G}$ of the cone $\hat{G} = \{v\} \star G$ branched over v;
- the homology group $H_1(M_n, \mathbb{Z})$ corresponds to the Jacobian group $\mathrm{Jac}(\hat{G})$.

[Gor71] C.M. Gordon, *A short proof of a theorem of Plans on the homology of the branched cyclic coverings of a knot*, Bull. Amer. Math. Soc. **77** (1971), pp. 85–87.

[MM23] A.D. Mednykh and I.A. Mednykh, *Cyclic coverings of graphs. Counting rooted spanning forests and trees, Kirchhoff index, and Jacobians*, Russian Math. Surveys **78**:3 (2023), pp. 501–548.

[Pla53] A. Plans, *Aportación al estudio de los grupos de homología de los recubrimientos cíclicos ramificados correspondientes a un nudo*, Rev. Acad. Ci. Madrid **47** (1953), pp. 161–193.

[Ste96] W. H. Stevens, *On the homology of branched cyclic covers of knots*, PhD thesis, Louisiana State Univ. and Agricultural & Mechanical College, 1996.

03.10
15:00–15:50

Is every knot isotopic to the unknot?

Sergey Melikhov

Steklov Mathematical Institute

50 years ago D. Rolfsen asked two questions[Rol74]:
(A) Is every knot in S^3 isotopic (= homotopic through embeddings) to a PL knot (or, equivalently, to the unknot)? In particular, is the Bing sling isotopic to a PL (= piecewise linear) knot?
(B) If two PL links in S^3 are isotopic, are they PL isotopic?

We show that the answer to (B) is positive if finite type invariants separate PL links in S^3 [Mel24b].

Regarding (A), it was previously shown by the author that not every link in S^3 is isotopic to a PL link [Mel21]. Now we show that the Bing sling is not isotopic to any PL knot by an isotopy which extends to an isotopy of 2-component links with linking number 1. Moreover, the additional component may be allowed to self-intersect, and even to get replaced by a new one as long as it represents the same conjugacy class in $G/[G', G'']$, where G is the fundamental group of the complement to the original component [Mel24a]. The proofs are based in part on a formula explaining the geometric meaning of the formal analogues of Cochran's derived invariants for PL links of linking number 1. These formal analogues are defined by using the 2-variable Conway polynomial [Mel24c].

[Mel21] S.A. Melikhov, *Topological isotopy and Cochran's derived invariants*, Topology, Geometry, and Dynamics: V.A. Rokhlin Memorial, Contemp. Math. Providence, RI: American Mathematical Society, 2021, pp. 249–266, arXiv: 2011.01409 [math.GT].

[Mel24a] S.A. Melikhov, *Is every knot isotopic to the unknot?* (2024), arXiv: 2406.09365 [math.GT].

[Mel24b] S.A. Melikhov, *Topological isotopy and finite type invariants* (2024), arXiv: 2406.09331 [math.GT].

[Mel24c] S.A. Melikhov, *Two-variable conway polynomial and Cochran's derived invariants* (2024), arXiv: math/0312007 [math.GT].

[Rol74] D. Rolfsen, *Some counterexamples in link theory*, Canad. J. Math. **26** (1974), pp. 978–984.

Geometric properties of surgery graphs in low-dimensional topology

01.10
17:00–17:25

Alexey Miller

Euler International Mathematical Institute

For the last twenty years, the question of the local and global geometric behavior of transformation graphs of various low-dimensional objects has received special attention. In this talk we will discuss a number of results obtained in this direction, namely, we will talk about the geometry of Gordian graphs of knot transformations and the structure of the big Dehn surgery graph.

Partial tribrackets of knots in thickened surfaces

04.10
12:50–13:40

Igor Nikonov

Lomonosov Moscow State University

We define a modification of Niebrzydowski tribracket construction for knots in a fixed thickened surface and give several examples of this invariant.

Augmentations and exact Lagrangian surfaces

01.10
17:30–17:55

Yu Pan

Tianjin University

Exact Lagrangian surfaces are important objects in the derived Fukaya category. Augmentations are objects of the augmentation category, which is the contact analog of the Fukaya category. In this talk, we discuss various relations between augmentations and exact Lagrangian surfaces. In particular, we realize augmentations, which is an algebraic object fully geometrically via exact Lagrangian surfaces.

01.10
16:00–16:25

Intersections of dual SL3-webs

Zhe Sun

University of Science and Technology of China

Fock and Goncharov introduced a pair of mirror moduli spaces associated to G and G^L which generalized the Teichmüller space and the decorated Teichmüller space, and they proposed a duality: the canonical basis of the regular function ring of one space X is parameterized by the tropical integral points of its mirror X^V. In this talk, I will explain my joint work with Linhui Shen and Daping Weng for SL3, where we introduce the topological asymmetric intersection numbers between webs on the surfaces to provide the duality pairings and the map from webs to tropical points. We prove that the map is the same as the previous one obtained by Douglas and myself. We relate the cluster algebra and skein algebra by this intersection number and prove the mutation equivariance, where the flip equivariance is a consequence.

04.10
11:50–12:40

Reflection equation algebras, quantum Toda system and quasideterminants

Dmitry Talalaev

Lomonosov Moscow State University

The theory of quantum groups is closely related to low-dimensional topology, and the whole field of quantum invariants arose due to the possibility of using the quantum R-matrix and the corresponding transfermatrix to construct such invariants. The transfer matrix is closely related to RTT algebra concept. In my report, I will talk about another family of quantum algebras, the so-called Reflection equation algebras. These algebras appeared in 1984 in a paper of Cherednik in describing scattering of systems with a boundary. I will talk on possible variants of topological problems in which such algebras can be used, and about the construction of a commutative family in RE-algebras using Gelfand-Retach quasi-determinants. This commutative family is a quantization of the complete Toda system and defines invariant functions on the higher Bruhat cell of RE-algebra.

Spatial graphs and associated links

01.10
10:30–11:20

Andrei Vesnin
Sobolev Institute of Mathematics & Tomsk State University

Spatial graphs are embeddings of graphs in three-dimensional space. The study of spatial graphs uses both combinatorial and topological methods. Two spatial graphs are said to be equivalent if there exists an ambient isotopy of space that transforms one spatial graph into another. Since an embedding of any cycle of a graph gives a knot, the theory of spatial graphs is a natural generalization of knot theory.

We will discuss basic properties of spatial graphs and corresponding knots and links. We will present relations between polynomial invariants of some classes of spatial graphs and Jones polynomial of corresponding knots and links.

The talk is based on a joint work with Olga Oshmarina, see arXiv.2404.12264.

On links in Poincaré homology sphere

04.10
16:00–16:25

Bao Vuong
Tomsk State University

We study links in Poincaré sphere. It is well-known that the Poincaré sphere can be obtained by doing surgery on left-handed trefoil knot in 3-sphere S^3 with framing -1. Thus, we represent a link in Poincaré sphere as a mix link diagram with a surgery component is left-handed trefoil. Further we get a presentation of fundamental group of link complement in Poincaré sphere and study classic invariants, related to it such as Alexander matrix, Seifert form, Alexander polynomial.

Heegaard Floer homology and the fundamental group

01.10
11:50–12:40

Jiajun Wang
Peking University

By the geometrization theorem, the fundamental group determines an irreducible three-manifold except lens spaces. It follows that the

Heegaard Floer homology of a three-manifold (or a null homologous knot) is determined by its fundamental group. A direct relationship between the fundamental group and Heegaard Floer homology is expected. We show that the hat version knot Floer homology of a $(1, 1)$ knot is determined by certain presentations of its fundamental group. This is joint work with Matthew Hedden and Xiliu Yang.

01.10
15:00–15:50

Growth tightness of quotients by confined subgroups

Wenyuan Yang

Peking University

A finitely generated group is said to have growth tightness if any quotient by an infinite normal subgroup has growth rate strictly less than that of the group. This property was introduced by Grigorchuk and de la Harpe, and was studied in various classes of negatively curved groups. Confined subgroups are a generalization of normal subgroups. In this talk, we establish the growth tightness of the quotient by confined subgroups in groups admitting the statistically convex-cocompact action with contracting elements. The result is sharp in the sense that the actions could not be replaced by actions with purely exponential growth. Applications to uniformly recurrent subgroups are discussed.

This is based on a joint work with Lihuang Ding.

04.10
15:00–15:50

Surfaces in Seifert manifolds

Xuezhi Zhao

Capital Normal University

In this talk, we illustrate kinds of Gröbner-Shirshov bases for the fundamental groups of some Seifert manifolds. As an application, we can decide homomorphisms from non-orientable surface groups to the fundamental groups of some Seifert manifolds. This is a joint work with Liao

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