





6th International Conference

GROUPS AND QUANDLES IN LOW-DIMENSIONAL TOPOLOGY



Program and Abstracts

June 10-11, 2025 Tomsk

6 INTERNATIONAL CONFERENCE

«GROUPS AND QUANDLES IN LOW-DIMENSIONAL TOPOLOGY»

Organizing Committee

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PROGRAM

June 10, Tuesday

Conference venue: Small conference room (1st floor), Research Library of Tomsk State University (34a Lenina ave., Tomsk)

Google Meet: https://meet.google.com/oka-bvdq-gzw

Schedule of talks according UTC+7 / GMT+7 time zone

9:00 - 9:20	Registration
9:20 - 9:30	Opening
9:30 – 10:15	Victor Buchstaber, n -valued groups, K ronecker sums and (x, y, z) -Wendt matrices
10:20 - 11:05	Andrei Vesnin, Amalgamations along surfaces with boundary in a handlebody
11:05 – 11:25	Coffee break
11:25 – 12:10	Andrey Vasil'ev, On multivalued groups of order 3
12:15 – 13:00	Andrei Malyutin, Fundamental groups and minimal knot diagrams
13:05 – 13:35	Mikhail Neshchadim, Triangular and near-trivial quandles
13:35 – 15:00	Lunch
15:00 – 15:30	Nikolai Erokhovets, Geometric hyperelliptic manifolds and hamiltonian subcomplexes in right-angled polytopes
15:35 – 16:05	Matvey Zonov, Growth of multivalued dynamics
16:10 – 16:40	Tatyana Kozlovskaya, Multi-virtual braid groups
16:40 – 17:00	Coffee break
17:00 – 17:20 (online)	Bao Vuong, Knot concordance in homology 3-spheres
17:25 – 17:45 (online)	Komal Negi, Classification of virtual links by arc shift move



PROGRAM

June 11, Wednesday

Conference venue: Small conference room (1st floor), Research Library of Tomsk State University (34a Lenina ave., Tomsk)

Google Meet: https://meet.google.com/oka-bvdq-gzw

Schedule of talks according UTC+7 / GMT+7 time zone

9:00 – 9:45	Taras Panov, Polyhedral products, graph products and p-central series
9:50 – 10:35	Dmitry Millionshchikov, Euler characteristics of positively graded Lie algebras and combinatorial identities
10:35 – 10:55	Coffee break
10:55 – 11:40	Olga Pochinka, On Morse-Smale diffeomorphisms with fixed points of pairwise different indices on 3 -manifolds
11:45 – 12:30	Evgeny Fominykh, Almost tetrahedral manifolds
12:35 – 13:05	Egor Timoshenko, Endomorphisms of a finite primary Abelian group as formal matrices
13:05 – 14:30	Lunch
14:30 – 15:00	Nikolay Abrosimov, On the volume of a non-Euclidean tetrahedron
15:05 – 15:35	Andrei Egorov, Right-angled hyperbolic polyhedra, hypererbolic links and vol-det conjecture
15:40 – 16:10	Maxim Ivanov, Non-abelian tensor product and orderability of groups
16:10 – 16:30	Coffee break
16:30 – 16:50 (online)	Pravin Kumar, Twisted Rokhlin Property for Mapping Class Groups
16:55 – 17:15 (online)	Deepanshi Saraf, (Co)homology of symmetric quandles over homogeneous Beck modules
17:20 – 17:40 (online)	Lokenath Kundu, z -Classes in the palindromic automorphism group of free groups
17:45 – 18:05 (online)	Debattam Das, Reversibility in the fundamental group of Seifert-fibered spaces



ABSTRACTS OF JUNE 10 TALKS

Victor Buchstaber

Algebraic n-valued groups, Kronecker sums and (x, y, z)-Wendt matrices

The talk is devoted to the well-known problem on the structure of integer polynomials $p_n(z,x,y)$ defining the laws of multiplication in n-valued groups G_n on the field of complex numbers \mathbb{C} . It will be shown that n-valued multiplication in the group G_n is realized in terms of the eigenvalues of the Kronecker sum of accompanying Frobenius matrices for polynomials $t^n - x$ of the variable t. We introduce the concept of the (x, y, z) - Wendt matrix. For $x = (-1)^n$, y = z = 1, we obtain the well-known Wendt matrix, the results on the determinant of which are used in number theory in connection with Fermat's Great Theorem.

Our key result I: for each natural n, the polynomial p_n is given by the determinant of the (x, y, z) - Wendt matrix.

We will discuss the concept of classes of symmetric n-algebraic n-valued groups. For each n, the group G_n belongs to one of these classes.

Our key result II: for n = 2 and 3, a description of universal objects of these classes is obtained.

The talk is based on the results obtained recently in collaboration with M.I. Kornev.

Andrei Vesnin

Amalgamations along surfaces with boundary in a handlebody

Let M be a connected orientable 3-manifold and F a compact connected orientable surface properly embedded in M. If F cuts M into two connected 3-manifolds X and Y, then we say that M is an amalgamation of X and Y along F; and if F cuts M into a connected 3-manifold X, then we say that M is a self-amalgamation of X along F. Due to Heegaard (1898) and Moise (1952) any closed connected orientable 3-manifold admits a Heegaard splitting that is an amalgamation of two handlebodies along a boundary.

In the talk we address the following question: when do amalgamations of handlebodies become a handlebody?

The talk is based on the following papers:

- [1] F. Lei, H. Liu, F. Li, A. Vesnin, A necessary and sufficient condition for a surface sum of two handlebodies to be a handlebody. Science China Mathematics, 63 (2020), 1997–2004.
- [2] S. Ding, F. Lei, W. Lin, A. Vesnin, Amalgamations along surfaces with boundary in a handlebody. Preprint (2025).

Andrey Vasil'ev On multivalued groups of order 3

We present a construction of multivalued groups of order 3 from strongly regular graphs and give a complete classification of the coset multivalued groups of order 3. The proof is based on the classification of rank 3 permutation groups having regular normal subgroups.

Andrei Malyutin

Fundamental groups and minimal knot diagrams

We will discuss some properties of classical knot diagrams related to several open conjectures in knot theory (including the conjecture on additivity of the crossing number of knots under connected sum and the conjecture that the crossing number of a satellite knot is not less than that of its companion). In particular, we will discuss relations between properties of minimal knot diagrams, geometric structure of knot complement, and Wirtinger presentations.

Mikhail Neshchadim

Triangular and near-trivial quandles

In this paper, we introduce and study classes of triangular and near-trivial quandles. We characterize these classes in terms of inner automorphisms. We prove that 1. every triangular quandle is almost trivial; 2. the class of near-trivial quandles is an algebraic variety; 3. the class of triangular quandles is not closed under taking Cartesian products.

We find necessary and sufficient conditions under which a generalized Alexander quandle is near-trivial or triangular. We also consider the questions of extension and constructive construction of such quandles.

In particular, we present an algorithm that allows us to construct all triangular quandles on a finite set of elements.

The work was supported by the Fundamental Research Program of the Siberian Branch of the Russian Academy of Sciences (no. I.1.5, Project FWNF 2022-0009).

Borodin A.N., Neshchadim M.V. and Simonov A.A. Triangular and near-trivial quandles. Submitted to the journal Algebra and logic.

Nikolai Erokhovets

Geometric hyperelliptic manifolds and hamiltonian subcomplexes in right-angled polytopes

A manifold M^n is called hyperelliptic if it has a hyperelliptic involution, i.e. an involution with orbit space S^n . Using the notions of a Hamiltonian cycle, theta-subgraph, and K_4 -subgraph on a right-angled 3-polytope, A.D. Mednykh and A.Yu. Vesnin [3] constructed examples of hyperelliptic 3-manifolds in the geometries \mathbb{R}^3 , S^3 , \mathbb{L}^3 , $\mathbb{L}^2 \times \mathbb{R}$, and $S^2 \times \mathbb{R}$. We generalize this construction to the n-dimensional case: we introduce the notion of a Hamiltonian $\mathcal{C}(n,k)$ -subcomplex in the boundary of a simple n-polytope with m facets and show that each such subcomplex Γ corresponds to a subgroup of rank m-k-1 in \mathbb{Z}_2^m acting freely on the real moment-angle manifold $\mathbb{R}\mathcal{Z}_p$ whose orbit space $N(P,\Gamma)$ is a manifold glued from 2^{k+1} copies of P. The group \mathbb{Z}_2^{k+1} acts on $N(P,\Gamma)$ and contains a hyperelliptic involution. We investigate for which n-dimensional geometries that are products of Euclidean spaces \mathbb{R}^k , spheres S^l and Lobachevsky spaces \mathbb{L}^r there exists a right-angled polytope with a Hamiltonian $\mathcal{C}(n,k)$ -subcomplex, and for n=4 we give a complete answer to this question [1,2].

References:

- [1] N. Erokhovets, Manifolds realized as orbit spaces of non-free Zk2-actions on real moment-angle manifolds, Proc. Steklov Inst. Math., 326 (2024), 177–218, arXiv:2403.00492v1.
- [2] N. Erokhovets, Four-manifolds defined by vector-colorings of simple polytopes, arXiv:2407.20575v1.
- [3] A.Yu. Vesnin, A.D. Mednykh. Three-dimensional hyperelliptic manifolds and Hamiltonian graphs, Siberian Math. J., 40:4 (1999), 628–643.

Matvey Zonov

Growth of multivalued dynamics

In this study we explore growth functions of multivalued groups and related multivalued dynamics. Upper bounds have been obtained on growth of multivalued dynamics of cyclic

coset groups, and on multivalued dynamics of cyclic abelian 2-valued groups. Cayley graphs of multivalued groups have been defined, and their elementary properties studied.

Tatyana Kozlovskaya Multi-virtual braid groups

We study multi-virtual braid groups which were introduced by Prof. Kauffman in 2024. We define some of their subgroups and construct representations of multi-virtual braid groups by automorphisms of some groups.

This is a recent joint work with Valeriy Bardakov, Komal Negi, and Madeti Prabhakar.

Bao Vuong

Knot concordance in homology 3-spheres

We proved the following theorem

Theorem. Let M, M' be homological spheres. Let W be a cobordism between M and M', and the boundary of W is disjoint union $\partial W = M \cup M'$. More over the inclusions $M \hookrightarrow W$ and $M' \hookrightarrow W$ induce isomorphisms on homology. Let k and k' be knots in M and M' correspondingly. If there exist a concordance $g: S^1 \times I \to W$ between k and k'. Then the Alexander polynomials of the knots k and k' are related by the following equation

$$\Delta_k(t) = p(t)p(1/t)\Delta_{k'}(t)$$

where $\Delta_k(t)$, $\Delta_{k'}(t)$ are the Alexander polynomials in t of the knots k, k' respectively and p(t) is a polynomial with integer coefficients.

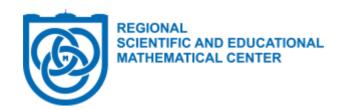
Komal Negi

Classification of virtual links by arc shift move

In virtual knot theory, the local move, arc shift move is an unknotting operation. However, this is not the case for virtual link theory.

In this talk, we establish that the arc shift operation on an n-component virtual link diagram acts as an unknotting operation when the virtual link is n-homogeneous proper, aiding in the classification of n-component virtual links up to arc shift equivalence. We explore the connection between the arc shift number and the odd writhe of homogeneous proper virtual links. Additionally, we identify sequences of virtual link diagrams L_n for which the upper bound of the arc shift number is exactly n.

This is collaborative work with Aastha Sahore, Amrender Singh Gill, Madeti Prabhakar.



ABSTRACTS OF JUNE 11 TALKS

Taras Panov

Polyhedral products, graph products and p-central series

We relate polyhedral products of topological spaces to graph products of groups. The loop homology algebras of polyhedral products are identified with the universal enveloping algebras of the Lie algebras associated with central series of graph products. By way of application, we describe the restricted Lie algebra associated with the lower **2**-central series of a right-angled Coxeter group and identify its universal enveloping algebra with the loop homology of the Davis-Januszkiewicz space.

Based on a joint work with Temurbek Rahmatullaev.

Dmitry Millionshchikov

Euler characteristics of positively graded Lie algebras and combinatorial identities

The talk will be devoted to generating series of Euler characteristics of positively graded Lie algebras. The famous Euler pentagonal identity can be considered as a consequence of the coincidence of two expressions for the (graded) Euler characteristic of the positive part of the Witt algebra [5]. In the finite-dimensional case, instead of the series, a polynomial arises (the so-called polynomial Euler characteristic according to [1]). We will discuss solved and unsolved problems related to the finite-dimensional version of Euler's pentagonal identity [2, 3, 4].

References:

- [1] V. Buchstaber. Polynomial Eulerian characteristic of nilmanifolds}, Funct. Anal. Appl., 58:1 (2024), 17-33.
- [2] Fialowski A., Millionshchikov D.V., Cohomology of graded Lie algebras of maximal class, Journal of Algebra, 296:1 (2007), 157-176.
- [3] D.V. Millionshchikov, Cohomology of Nilmanifolds and Gontcharova's Theorem, CONM, 288, AMS (2001), 381-385.

- [4] D.V. Millionshchikov, Algebra of formal vector fields on the line and Buchstaber's conjecture, Funct. Anal. Appl., 43:4 (2009), 264-278
- [5] D.B. Fuks, Cohomology of infinite-dimensional Lie algebras, Consultants Bureau, New York, 1986.

Olga Pochinka

On Morse-Smale diffeomorphisms with fixed points of pairwise different indices on 3-manifolds

In the works of V. Grines, V. Medvedev, E. Zhuzhoma, O. Pochinka and E. Talanova, Morse-Smale diffeomorphisms f with chain recurrent points of pairwise different indices defined on closed 3-manifolds M^3 were considered. They established that M^3 is homeomorphic to the lens space L(p,q) and the wandering set of f contains at least p noncompact heteroclinic curves. If M^3 is non-orientable then it is skew product of S^2 and S^1 . Countable families of pairwise non-conjugate diffeomorphisms without heteroclinic curves are constructed, with a wild embedding of all saddle separatrices at once, as well as with a tame embedding, in which a pair of one-dimensional separatrices forms a mildly wild frame. The constructed examples differ fundamentally from the known Pixton diffeomorphisms, in which at least one saddle separatrix is always tame.

Evgeny Fominykh Almost tetrahedral manifolds

One of the most interesting invariants of compact 3-manifolds with boundary is the triangulation complexity, i.e. the number of tetrahedra in its minimal ideal triangulation. Upper bounds on the complexity usually arise from explicit construction of triangulations, while finding lower bounds is generally a difficult problem. A well-known result is a lower bound on the triangulation complexity in terms of the hyperbolic volume of the manifold. In this talk, we will define almost tetrahedral manifolds and establish the complexity of any almost tetrahedral manifold. We will also present a method for constructing infinite series of almost tetrahedral manifolds. In particular, this will allow us to construct some infinite series of link complements in the 3-sphere of known triangulation complexity.

Egor Timoshenko

Endomorphisms of a finite primary Abelian group as formal matrices

The endomorphisms of a finite primary Abelian group are in one-to-one correspondence with the elements of a suitable set of formal matrices which we denote by R. We endow this

set R with a multiplication such that R becomes a ring which is isomorphic to the endomorphism ring of our primary group. For matrices in R, we define the determinant which is a multiplicative map with a residue class ring as its codomain. We prove that a matrix in invertible (left invertible, right invertible) in R if and only if its determinant is invertible. We also give an algorithm for finding an inverse matrix in R.

Nikolay Abrosimov

On the volume of a non-Euclidean tetrahedron

The talk will give an overview of the latest results on finding exact formulas for calculating the volumes of non-Euclidean tetrahedra. The classical formula of G. Sforza [1] expresses the volume of a compact hyperbolic tetrahedron in terms of dihedral angles. Its modern proof is proposed in [2] where a version of Sforza's formula for the volume of a spherical tetrahedron is also given. The formula in terms of edge lengths is obtained in the joint work of the author with B. Vuong [3]. The known volume formulas for a hyperbolic tetrahedron of a general type are very complicated, so natural question arises: to find more simple formulas for sufficiently wide families of hyperbolic tetrahedra.

We will consider hyperbolic tetrahedra of special types: ideal, biorthogonal, trirectangular and their generalizations. The volumes of an ideal and biorthogonal tetrahedron were known to N.I. Lobachevsky. We will present new formulas for calculating volumes and normalized volumes of a hyperbolic trirectangular tetrahedron [4] as well as its generalization for 4-parameter family of tetrahedra with one edge orthogonal to the face. The latter formulas can be used to calculate the volumes of more complex polyhedra in the Lobachevsky space. At the end of the talk we will present a new formula for the volume of a spherical trirectangular tetrahedron [5].

References:

- [1] G. Sforza Spazi metrico-proiettivi // Ricerche di Estensionimetria Integrale Ser. III VIII(Appendice) 1907 P. 41–66.
- [2] N.V. Abrosimov A.D. Mednykh Volumes of polytopes in constant curvature spaces //Fields Inst. Commun. 2014 V. 70 P. 1–26. arXiv:1302.4919
- [3] N. Abrosimov B. Vuong Explicit volume formula for a hyperbolic tetrahedron in terms ofedge lengths // Journal of Knot Theory and Its Ramifications 2021 V. 30 No. 10 2140007.arXiv:2107.03004
- [4] N. Abrosimov S. Stepanishchev The volume of a trirectangular hyperbolic tetrahedron //Siberian Electronic Mathematicsl Reports 2023 V. 20 No. 1 P. 275–284.
- [5] N. Abrosimov B. Bayzakova The volume of a spherical trirectangular tetrahedron //Siberian Electronic Mathematical Reports (in print).

Andrei Egorov

Right-angled hyperbolic polyhedra, hypererbolic links and vol-det conjecture

In this talk, we will discuss how volume bounds for ideal hyperbolic polyhedra and hyperbolic links allows us to expend the class of hyperbolic links for which the volume-determinent is known to hold. In particular, we consider links with highly twisted diagrams and demonstrate how improved upper bounds on their volumes, combined with determinant inequalities, lead to family of links satisfying the conjecture.

Maxim Ivanov

Non-abelian tensor product and orderability of groups

In 1987 R. Brown and J.-L. Loday defined non-abelian tensor product of groups. The construction arises in applications of a generalized Van Kampen theorem. In particular, third homotopy group of the suspension of an Eilenberg–MacLane space K(G, 1) is isomorphic to a kernel of a natural map $G \otimes G \to G$. We will discuss properties, such as orderability of tensor products, and their relation to those of the initial groups.

Pravin Kumar

Twisted Rokhlin Property for Mapping Class Groups

A topological group G is said to have the Rokhlin property if it has a dense conjugacy class. Taking motivation from this, we can define the ϕ -twisted conjugacy class of an element $h \in G$ as the set $\{gh\phi(g)^{-1} | g \in G\}$ for $\phi \in Aut(G)$ and ask whether a topological group has a dense ϕ -twisted conjugacy class. In this talk, we will answer this question when G is a mapping class group of a surface with empty boundary.

This supplements the recent work of Lanier and Vlamis on the Rokhlin property of mapping class groups. This is part of a joint work with Dr. Apeksha Sanghi and Dr. Mahender Singh.

Deepanshi Saraf

(Co)homology of symmetric quandles over homogeneous Beck modules

A quandle equipped with a good involution is referred to as symmetric. It is known that the cohomology of symmetric quandles gives rise to strong cocycle invariants for classical and surface links, even when they are not necessarily oriented. In this talk, I will introduce the category of symmetric quandle modules and will see that these modules completely determine the Beck modules in the category of symmetric quandles. Consequently, this

establishes suitable coefficient objects for constructing appropriate (co)homology theories. We develop an extension theory of modules over symmetric quandles and propose a generalized (co)homology theory for symmetric quandles with coefficients in a homogeneous Beck module, which also recovers the symmetric quandle (co)homology developed by Kamada and Oshiro [Trans. Amer. Math. Soc. (2010)]. Our constructions also apply to symmetric racks.

Lokenath Kundu

z-Classes in the palindromic automorphism group of free groups

The concept of *z-classes*, introduced by Kulkarni, partitions a group according to conjugacy of centralizers and has become a useful tool in understanding the algebraic and dynamical structure of groups. In this talk, we study the *z*-classes in the palindromic automorphism group $\Pi A(F_n)$, a subgroup of the automorphism group of a free group $Aut(F_n)$ defined by automorphisms sending each generator to a palin- dromic word. We characterize the image of $\Pi A(F_n)$ under the abelian- ization map into $GL_n(\mathbb{Z})$, and using this structure, we show that $\Pi A(F_n)$ contains infinitely many z-classes. For n = 3, we classify the z-classes of various generators and analyze reducible palindromic automorphisms in terms of matrix conjugacy.

Debattam Das

Reversibility in the fundamental group of Seifert-fibered spaces

An element a in a group Γ is called reversible if there exists $g \in \Gamma$ such that $gag^{-1} = a^{-1}$. The reversible elements are also known as `real elements' or `reciprocal elements' in literature. In this work, we have classified the reversible elements in a Seifert-fibered group. This is joint work with Ms. Anushree Das.



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