



## **KNOTS, ALGEBRA AND GEOMETRY**

Online conference on the occasion of 60th birthdays of Valeriy Bardakov and Andrei Vesnin

The online meeting will celebrate Prof. Valeriy Bardakov and Prof. Andrei Vesnin turning 60 this year. The meeting will focus on interactions between topology, geometry and algebra, specifically, from the point of view of low dimensional topology, the topics in which both Valeriy and Andrei have profound contributions.

Key topics include but are not limited to the following:

- Geometric structures on manifolds and orbifolds
- Classical and virtual knots
- Algebraic structures in knot theory
- Applications of the theory of 3-manifolds

### **ORGANIZERS**

Nikolay Abrosimov (Sobolev Institute of Mathematics)

Krishnendu Gongopadhyay (IISER Mohali)

Tatyana Kozlovskaya (Tomsk State University)

Timur Nasybullov (Sobolev Institute of Mathematics)

Madeti Prabhakar (IIT Ropar)

Mahender Singh (IISER Mohali)

Web-page: <https://sites.google.com/view/knots-algebra-geometry>

## Friday, March 17

### Schedule of talks according GMT+7 time zone

**Zoom:** <https://us02web.zoom.us/j/83636573209?pwd=c2dwSml4cWpKZVdjY2tSdlpmNXIjQT09>

Conference ID: 836 3657 3209

Pass code: 752392

<b>10:30 – 11:10</b>	<b>Akio Kawauchi</b> <i>Ribbonness of Kervaire's sphere-link in homotopy 4-sphere and Whitehead aspherical conjecture</i>
<b>11:20 – 11:50</b>	<b>Jie Wu</b> <i>Homotopy Patterns and Cablings on Braids</i>
<b>11:50 – 17:20</b>	<b>Long break</b>
<b>17:20 – 17:50</b>	<b>Dmitry Talalaev</b> <i>Alternative algebraic structures in low-dimensional topology</i>
<b>18:00 – 18:40</b>	<b>Carlo Petronio</b> <i>Realizability of Hurwitz branch data with a length-2 partition</i>

## Saturday, March 18

### Schedule of talks according GMT+7 time zone

**Zoom:** <https://us02web.zoom.us/j/83636573209?pwd=c2dwSml4cWpKZVdjbj2tSdlpmNXIjQT09>

Conference ID: 836 3657 3209

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<b>09:50 – 10:20</b>	<b>Ualbai Umirbaev</b> <i>Representations via differential algebras and equationally Noetherian algebras</i>
<b>10:30 – 11:10</b>	<b>Louis H. Kauffman</b> <i>Mock Alexander Polynomials</i>
<b>11:20 – 11:50</b>	<b>Mohamed Elhamdadi</b> <i>Invariants of knots and links from idempotents in quandle rings</i>
<b>11:50 – 16:40</b>	<b>Long break</b>
<b>16:40 – 17:10</b>	<b>Zoran P. Rakić</b> <i>On a model of nonlocal de Sitter Gravity and its solutions</i>
<b>17:20 – 17:50</b>	<b>Vladimir Vershinin</b> <i>On our joint works with Valerii on braids</i>
<b>18:00 – 18:40</b>	<b>Vladimir Turaev</b> <i>Quasi-Lie bialgebras of loops in quasi-surfaces</i>

## Sunday, March 19

### Schedule of talks according GMT+7 time zone

**Zoom:** <https://us02web.zoom.us/j/83636573209?pwd=c2dwSml4cWpKZVdjb2tSdlpmNXIjQT09>

Conference ID: 836 3657 3209

Pass code: 752392

<b>10:30 – 11:10</b>	<b>Seiichi Kamada</b> <i>On braid presentation of twisted links and spatial graphs</i>
<b>11:20 – 11:50</b>	<b>Manoj Yadav</b> <i>Skew Braces</i>
<b>11:50 – 16:40</b>	<b>Long break</b>
<b>16:40 – 17:10</b>	<b>Paolo Bellingeri</b> <i>Groups of cacti and planar braids</i>
<b>17:20 – 17:50</b>	<b>Vsevolod Gubarev</b> <i>Rota-Baxter groups and braces</i>
<b>18:00 – 18:40</b>	<b>Sophia Lambropoulou</b> <i>From plat closure to standard closure of braids and back</i>

## ABSTRACTS

### **Groups of cacti and planar braids**

**Paolo Bellingeri**

University of Caen Normandy

paolo.bellingeri@unicaen.fr

Already known as quasi-braid groups, Henriques and Kamnitzer defined cactus groups in terms of actions on tensor products in cobordism categories. If group presentations are known for these groups, their combinatorial properties and the relationships with braid groups are not yet fully understood.

In this talk we will explore some first results of a combinatorial nature.

Based on joint works with Hugo Chemin & Victoria Lebed and Eddy Godelle & Luis Paris.

### **Rota-Baxter groups and braces**

**Vsevolod Gubarev**

Sobolev Institute of Mathematics

wsevolod89@gmail.com

We study the connection between recently defined Rota-Baxter groups and skew left braces, generalizations and applications of this connection.

### **Invariants of knots and links from idempotents in quandle rings**

**Mohamed Elhamdadi**

University of South Florida

emohamed@usf.edu

We will discuss some new results about idempotents in quandle rings. We will show how idempotents can be used to obtain invariants of knots and links. The results are based on joint works with Brandon Nunez, Mahender Singh and Swain Dipali.

### **On braid presentation of twisted links and spatial graphs**

**Seiichi Kamada**

Osaka University

kamada@math.sci.osaka-u.ac.jp

Twisted links are defined by using link diagrams on a plane possibly with virtual crossings and bars modulo a generalization of Reidemeister moves, and they correspond to stable equivalence classes of link in oriented thickenings of surfaces. We discuss Alexander and Markov theorem for twisted links. This is a joint work with Madeti Prabhakar and Komal Negi. And we also introduce Alexander and Markov theorem for spatial graphs. This is bases on a joint research work with V. Lebed during her visit to Osaka as JSPS post-doc fellow.

**Mock Alexander Polynomials**  
**Louis H. Kauffman**  
University of Illinois at Chicago  
kauffman@uic.edu

This talk will discuss generalizations of the Alexander-Conway polynomial to starred knots (starred regions correspond to holes in a thickened surface), knotoids, virtual knots and knots in thickened surfaces. The Mock Alexander Polynomials can be defined on diagrams whenever the number of faces in the diagram is equal to the number of crossings. Our generalizations use state summations that can be expressed in terms of permanents of matrices associated with the diagrams of the starred entities. These state summations generalize structures in the author's monograph Formal Knot Theory giving state models for the Alexander-Conway polynomial. We will show by examples how the state sums we are using go beyond the determinant formulations of the original Alexander polynomial and will give a number of new examples of computations and formulations of the invariants. We discuss the relationships of skein theory, state sums, classical models and conjectured generalizations of the Fox-Milnor theorem for these invariants.

This talk is joint work with Neslihan Gugumcu.

**Ribbonness of Kervaire's sphere-link in homotopy 4-sphere  
and Whitehead aspherical conjecture**

**Akio Kawauchi**  
Osaka Central Advanced Mathematical Institute,  
Osaka Metropolitan University  
kawauchi@omu.ac.jp

Kervaire's spherical link in homotopy 4-sphere is identified with a ribbon sphere-link in 4-sphere. By developing this idea, Whitehead aspherical conjecture is claimed to be true.

**From plat closure to standard closure of braids and back**

**Sofia Lambropoulou**  
National Technical University of Athens  
sofia@math.ntua.gr

Given a knot or link in the form of the plat closure of a braid, we describe an algorithm to obtain a braid representing the same link with the standard closure, and vice-versa. We analyze the three cases of links: in  $\mathbb{R}^3$ , in handlebodies and in thickened surfaces. We show that the algorithm is quadratic in the number of crossings and loop generators of the braid when passing from plat to standard closure, while it is linear when passing from standard to plat closure. Each type of closure has its advantages. For example the plat closure is most suitable when representing a c.c.o. 3-manifold by its handle decomposition, whilst the standard closure is most suitable when representing a c.c.o. 3-manifold by a surgery description.

This is joint work with Paolo Cavicchioli (U Modena).

## Realizability of Hurwitz branch data with a length-2 partition

Carlo Petronio

University of Pisa

carlo.petronio@unipi.it

To a branched cover  $f : \tilde{\Sigma} \rightarrow \Sigma$  between closed surfaces one can associate a combinatorial datum given by the surfaces themselves up to homeomorphism, the degree  $d$  of  $f$ , the number  $n$  of branching points of  $f$  and the partitions  $\{\pi_i = [d_{ij}]_{j=1}^{m_i}\}_{i=1}^n$  of  $d$  given by the local degrees of  $f$  at the preimages of the branching points. This datum must satisfy the Riemann-Hurwitz condition plus some extra ones if either  $\tilde{\Sigma}$  or  $\Sigma$  or both are non-orientable. A very old question of Hurwitz asks whether a combinatorial datum satisfying the necessary conditions is actually realizable (namely, associated to some existing  $f$ ) or not (in which case it is called exceptional). The answer is now known in a vast number of cases. In particular, it is always in the affirmative if  $\Sigma$  is orientable and not the sphere. Moreover, the case of a non-orientable  $\Sigma$  can be reduced to the case of an orientable one. And the answer is also affirmative if  $\Sigma$  is the sphere and one of the partitions  $\pi_i$  has length 1. We will concentrate here on the case where  $\Sigma$  is the 3-sphere and one of the partitions  $\pi_i$  has length 2. A complete answer in this case was provided by Pakovich when  $\tilde{\Sigma}$  is also the sphere, and we will give a thorough solution for the case of an arbitrary  $\tilde{\Sigma}$ . Our argument uses Pakovich's statement and an induction on both the genus of  $\tilde{\Sigma}$  and the number  $n$  of branching points. This is joint work with Filippo Gianni Baroni.

## On a model of nonlocal de Sitter Gravity and its solutions

Zoran P. Rakić

University of Belgrade

zoran.rakic@matf.bg.ac.rs

Despite to all significant gravitational phenomena discovered and predicted by general theory of relativity, it is not a complete theory. One of actual approaches towards more complete theory of gravity is its nonlocal modification.

We consider nonlocal modification of the Einstein theory of gravity in framework of the pseudo-Riemannian geometry. The nonlocal term has the form  $H(R)F(\square)G(R)$ , where  $H$  and  $G$  are differentiable functions of the scalar curvature  $R$ , and  $F(\square) = \sum_{n=0}^{\infty} f_n \square^n$  is an analytic function of the d'Alembert operator  $\square$ .

After consideration of several models of the above-mentioned type, here we deal with  $H(R) = G(R) = \sqrt{R - 2\Lambda}$ , and where  $F(\square)$  is an analytic function of the d'Alembert operator  $\square$  and also  $\square^{-1}$ . Specially, we searching several classes of scaling factors, and we find some new exact cosmological solutions. We are paid our attention to the scaling factor of the form  $a(t) = At^{\frac{2}{3}}e^{\frac{\Lambda}{14}t^2}$ , and we test validity of obtained solutions with experimental data and their interpretations.



## **Alternative algebraic structures in low-dimensional topology**

**Dmitry Talalaev**

Lomonosov Moscow State University and Yaroslavl State University

dtalalaev@yandex.ru

The field of quantum invariants of knots, which is an example of a very interesting modern cross-disciplinary mathematical theory, was motivated purely by topological problems, but stimulated significant progress in algebra and the theory of integrable systems. In addition to the theory of quantum groups based on the Yang-Baxter equation, the problem of quantum invariants has higher analogues associated with the higher  $n$ -simplex equations. In a broader sense, the theory of quantum invariants can be considered as a program for the algebraization of topological invariants. Along this path, a large number of interesting algebraic structures have already emerged, finding their place in algebra, topology and the theory of integrable systems. Such structures include: quandles, racks, braces, rack bialgebras, infinitesimal bialgebras.

In this overview report, I will talk about generalizations of the problem of quantum invariants to higher dimensions, as well as to the listed alternative algebraic structures. Some of the results presented were obtained jointly with V.G. Bardakov.

## **Quasi-Lie bialgebras of loops in quasi-surfaces**

**Vladimir Turaev**

Indiana University, Bloomington

vturaev@yahoo.com

A quasi-surface is obtained by gluing a surface  $\Sigma$  to an arbitrary topological space along a mapping of several disjoint subsegments of  $\partial\Sigma$  to that space. We define and investigate operations on free homotopy classes of loops in a quasi-surface similar to the standard Lie bracket and Lie cobracket of free homotopy classes of loops in a surface.

## **Representations via differential algebras and equationally Noetherian algebras**

**Ualbai Umirbaev**

Wayne State University

umirbaev@yahoo.com

We show that free algebras of the variety of algebras generated by the Witt algebra  $W_n$ , the left-symmetric Witt algebra  $L_n$ , and the symplectic Poisson algebra  $P_n$  can be described as subalgebras of differential polynomial algebras with respect to appropriately defined products. Using these representations, we prove that  $W_n$ ,  $L_n$ ,  $P_n$ , and the free algebras of the varieties of algebras generated by these algebras are equationally Noetherian.

This is joint work with A.A. Mikhalev and M. Mustafa

## **On our joint works with Valerii on braids**

**Vladimir Vershinin**

Montpellier 2 University

vlaversh@gmail.com

The talk is a survey of joint works starting with the paper of 2012 on Brunnian braids and till the last one of 2022 on homotopy braids.

## **Homotopy Patterns and Cablings on Braids**

**Jie Wu**

Beijing Institute of Mathematical Sciences and Applications

wujie@hebtu.edu.cn

The notion of homotopy pattern was recently introduced by Roman Mikhailov in the Proceedings of the ICM 2022. In this talk, we will give an introduction to the homotopy patterns in group theory proposed by Mikhailov as well as a recent solution of Laurent Bartholdi and Roman Mikhailov to a longstanding dimension problem in group theory using homotopy theory. Moreover, we will give a report of my joint works with Valeriy Bardakov, Volodia Vershinin and others on the topic such as cablings on braids.

## **Skew Braces**

**Manoj Yadav**

Harish-Chandra Research Institute Prayagraj

myadav@hri.res.in

We'll present some procedures of constructing skew left braces and its applications. Its connection with set-solutions of Yang-Baxter Equation / braided relation will be explained.

## PARTICIPANTS

Nikolay Abrosimov (Sobolev Institute of Mathematics)	nikolay.abrosimov@gmail.com
Valeriy Bardakov (Sobolev Institute of Mathematics)	bardakov@math.nsc.ru
Paolo Bellingeri (University of Caen Normandy)	paolo.bellingeri@unicaen.fr
Pragya Belwal (Indian Institute of Science Education and Research Mohali)	pragyabelwal@gmail.com
Bogdan Chuzhinov	b.chuzhinov@g.nsu.ru
Debattam Das (Indian Institute of Science Education and Research Mohali)	debattam123@gmail.com
Neeraj k. Dhanwani (Indian Institute of Science Education and Research Mohali)	neerajk.dhanwani@gmail.com
Sandipan Dutta (Indian Institute of Science Education and Research Mohali)	sandipandutta98@gmail.com
Andrei Egorov (Novosibirsk State University)	a.egorov2@g.nsu.ru
Mohamed Elhamedadi (University of South Florida)	emohamed@usf.edu
Krishnendu Gongopadhyay (Indian Institute of Science Education and Research Mohali)	krishnendu@iisermohali.ac.in
Liliya Grunwald (Novosibirsk State University)	lfb_o@yahoo.co.uk
Vsevolod Gubarev (Sobolev Institute of Mathematics)	vsevolodgu@mail.ru
Maxim Ivanov (Novosibirsk State University)	m.ivanov2@g.nsu.ru
Sahil Joshi (Indian Institute of Technology Ropar)	2018maz0001@iitrpr.ac.in
Seiichi Kamada (Osaka University)	kamada@math.sci.osaka-u.ac.jp
Biswadeep Karmakar (Indian Institute of Science Education and Research Mohali)	biswadeep.isi@gmail.com
Louis H. Kauffman (University of Illinois Chicago)	loukau@gmail.com
Kirandeep Kaur (Guru Nanak Dev University)	kirandeepoffical@gmail.com
Akio Kawauchi (Osaka Metropolitan University)	kawauchi@omu.ac.jp
Tatyana Kozlovskaya (Tomsk State University)	konus_magadan@mail.ru
Pravin Kumar (Indian Institute of Science Education and Research Mohali)	pravin444enaj@gmail.com

Sofia Lambropoulou (National Technical University of Athens)	sofia@math.ntua.gr
Tejbir Lohan (Indian Institute of Science Education and Research Mohali)	tejbirlohan70@gmail.com
Prabhakar Madeti (Indian Institute of Technology Ropar)	prabhakar@iitrpr.ac.in
Alexander Mednykh (Sobolev Institute of Mathematics)	smedn@mail.ru
Rahul Mondal (Indian Institute of Science Education and Research Mohali)	canvas.rahul@gmail.com
Abhishek Mukherjee (Kalna College)	abhimukherjee.math10@gmail.com
Tushar Kanta Naik (National Institute of Science Education and Research Bhubaneswar)	mathematics67@gmail.com
Neha Nanda (Universite de Caen Normandie)	nehananda94@gmail.com
Timur Nasybullov (Sobolev Institute of Mathematics)	timur.nasybullov@mail.ru
Komal Negi (Indian Institute of Technology Ropar)	komal.20maz0004@iitrpr.ac.in
Olga Oshmarina	o.oshmarina@g.nsu.ru
Carlo Petronio (University of Pisa)	carlo.petronio@unipi.it
Madeti Prabhakar (Indian Institute of Technology Ropar)	prabhakar@iitrpr.ac.in
Zoran P. Rakić (University of Belgrade)	zoran.rakic@matf.bg.ac.rs
Apeksha Sanghi (Indian Institute of Science Education and Research Mohali)	apekshasanghi93@gmail.com
Deepanshi Saraf (Indian Institute of Science Education and Research Mohali)	deepanshisaraf@gmail.com
Mahender Singh (Indian Institute of Science Education and Research Mohali)	mahender@iisermohali.ac.in
Manpreet Singh (Instituto Superior Técnico Lisbon)	manpreetsingh23128@gmail.com
Stepan Stepanishev (Novosibirsk State University)	eeeevae@ gmail.com
Dmitry Talalaev (Lomonosov Moscow State University and Yaroslavl State University)	dtalalaev@yandex.ru
Vladimir Turaev (Indiana University, Bloomington)	vturaev@yahoo.com
Ualbai Umirbaev (Wayne State University)	umirbaev@yahoo.com
Vladimir Vershinin (Montpellier 2 University)	vlaversh@gmail.com

Andrei Vesnin (Sobolev Institute of Mathematics, Tomsk State University and Novosibirsk State University)

vesnin@math.nsc.ru

Bao Vuong (Tomsk State University)

vuonghuubao@live.com

Jie Wu (Beijing Institute of Mathematical Sciences and Applications)

wujie@hebtu.edu.cn

Manoj Yadav (Harish-Chandra Research Institute Prayagraj)

myadav@hri.res.in