REGIONAL SCIENTIFIC AND EDUCATIONAL MATHEMATICAL CENTER

FACULTY OF MECHANICS AND MATHEMATICS

НАУКА И УНИВЕРСИТЕТЫ
НАЦИОНАЛЬНЫЕ ПРОЕКТЫ РОССИИ

5th International Conference

GROUPS AND QUANDLES IN LOW-DIMENSIONAL TOPOLOGY

Program and Abstracts

June 13-15, 2024
Tomsk

5 INTERNATIONAL CONFERENCE

«GROUPS AND QUANDLES IN LOW-DIMENSIONAL TOPOLOGY»

Organizing Committee

Andrei Vesnin	Sobolev Institute of Mathematics, Tomsk State University and Novosibirsk State University, Novosibirsk, Tomsk, Russia
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REGIONAL SCIENTIFIC AND EDUCATIONAL MATHEMATICAL CENTER

5 INTERNATIONAL CONFERENCE GROUPS AND QUANDLES IN LOW-DIMENSIONAL TOPOLOGY

PROGRAM

June 13, Thursday

Conference venue: Research Library of Tomsk State University (34a Lenina ave., Tomsk)

Google Meet: **https://meet.google.com/saq-dddq-gag**

Schedule of talks according UTC+7 / GMT+7 time zone

10:00 – 10:30	Andrei Vesnin, *On Yamada polynomial of spatial graphs and Jones polynomial of related links*
10:40 – 11:10 (online)	Vassily Manturov, *How to use virtual knot theory to upgrade braid group presentation*
11:20 – 11:40	Coffee break
11:40 – 12:10	Valeriy Bardakov, *Invariants of handlebody-links and spatial graphs*
12:20 – 12:50	Philipp Korablev, *Fibonacci representations of the braid group*
13:00 – 13:30	Tatyana Kozlovskaya, *Representations of the singular braid groups*
13:40 – 15:00	Lunch
15:00 – 15:30 (online)	Mahender Singh, *Separability properties of quandles*
15:40 – 16:10 (online)	Krishnendu Gongopadhyay, *Dehn function for the group of palindromic automorphisms*
16:20 – 16:40	Coffee break
16:40 – 17:10 (online)	Nikolai Erokhovets, *Manifolds realized as orbit spaces of non-free* Z_2^k*-actions on real moment-angle manifolds*
17:20 – 17:50 (online)	Igor Nikonov, *Biquandloids of knots in thickened surfaces*

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5 INTERNATIONAL CONFERENCE GROUPS AND QUANDLES IN LOW-DIMENSIONAL TOPOLOGY

PROGRAM

June 14, Friday

Conference venue: Research Library of Tomsk State University (34a Lenina ave., Tomsk)

Google Meet: **https://meet.google.com/saq-dddq-gag**

Schedule of talks according UTC+7 / GMT+7 time zone

10:00 – 10:30	Andrey Vasiliev, *Hypergroups, association schemes and Hall's theorems on finite solvable groups*
10:40 – 11:10	Evgeny Fominykh, *Minimal ideal triangulations of 3-manifolds*
11:20 – 11:40	Coffee break
11:40 – 12:10 (online)	Dmitri Talalaev, *Multivalued quandles: algebra and topology*
12:20 – 12:50	Maxim Ivanov, *Virtual knot groups and circular orderability*
13:00 – 13:30	Anton Kazakov, *The Kramers–Wannier duality as the Tutte type duality*
13:40 – 15:00	Lunch
15:00 – 15:30	Andrei Egorov, *Upper bounds for the volumes of hyperbolic polyhedra and hyperbolic links*
15:40 – 16:10 (online)	Andrei Malyutin, *Quantization effects in groups acting on the circle*
16:20 – 16:40	Coffee break
16:40 – 17:10 (online)	Komal Negi, *Singular twisted links and singular twisted virtual braids*
17:20 – 17:50 (online)	Kirandeep Kaur, *Virtual knots with unknotting index (n, m)*

REGIONAL SCIENTIFIC AND EDUCATIONAL MATHEMATICAL CENTER

5 INTERNATIONAL CONFERENCE GROUPS AND QUANDLES IN LOW-DIMENSIONAL TOPOLOGY

PROGRAM

June 15, Saturday

Conference venue: Research Library of Tomsk State University (34a Lenina ave., Tomsk)

Google Meet: **https://meet.google.com/saq-dddq-gag**

Schedule of talks according UTC+7 / GMT+7 time zone

10:00 – 10:30	Mikhail Neshchadim, *Constructions of quandles over groups and rings and some questions of group theory*
10:40 – 11:10	Nikolay Abrosimov, *Euclidean volume of a cone manifold over any hyperbolic knot is an algebraic number*
11:10 – 11:30	Coffee break
11:30 – 12:00	Bao Vuong, *On links in Poincare sphere*
12:10 – 12:40	Matvei Zonov, *On verbal solutions of the Yang-Baxter equation*
13:00 – 17:00	Excursion

ABSTRACTS OF JUNE 13 TALKS

Andrei Vesnin

On Yamada polynomial of spatial graphs and Jones polynomial of related links

We will discuss invariants of spatial embeddings of graphs into a 3-sphere. Since any simple cycle of a graph is embedding as a knot, the theory of spatial graphs can be considered as a generalization of the knot theory. Two spatial graphs are said to be equivalent if there is an ambient isotopy of the 3-sphere which transforms one spatial graph to another. As well as knots and links, spatial graphs can be studied from their diagrams and Seifert surfaces.

It was shown in [1] that an invariant link can be associated to an embedding a planar three-valent graph with at most six edges. The Yamada polynomial introduced in [2] is the most useful invariant of spatial graphs. A relation between the normalized Yamada polynomial of a spatial theta-graph and the Jones polynomial of the associated link was described in [3]. Let K4 be the complete graph with four vertices. We will present a relation between the normalized Yamada polynomials of a spatial K4-graph and its spatial subgraphs with the Jones polynomial of the associated link. The talk is based on the joint work with O. Oshmarina [4].

The research was supported by the Ministry of Science and Higher Education of Russia (agreement No. 075-02-2024-1437).

References

[1] L.H. Kauffman, J. Simons, K. Wolcott, P. Zhao, Invariants of theta-curves and other graphs in 3-space. Topology and Its Applications, 49 (1993), 193–216.

[2] S. Yamada, An invariant of spatial graphs. Graph Theory, 13 (1989), 537–531.

[3] Y. Huh, Yamada polynomial and associated link of theta-curves. Discrete Mathematics, 347:1 (2024), paper 113684.

[4] O. Oshmarina, A.Vesnin, Polynomials of complete spatial graphs and Jones polynomial of related links. Preprint arXiv:2404.12264 (2024), 28 pp.

Vassily Manturov

How to use virtual knot theory to upgrade braid group presentation

Any n-strand (pure) braid can be turned into a braid on the cylinder if one fixes one strand as an axis. It turns out that braids on the cylinder have hidden structure of "virtual crossings" which allows one to map classical braids to the object called "flat virtual braids". The latter enjoy many properties from virtual knot theory, in particular, one can use new variables coming from virtual crossings to enrich classical knot invariants. We shall concentrate on an enhancement of the Burau representation.

This is a joint work with I.M. Nikonov. The talk is based on the paper: Russian Mathematical Surveys, 78, 2 (2023), 393–395

Valeriy Bardakov

Invariants of handlebody-links and spatial graphs

A G-family of quandles is an algebraic construction which was proposed by A. Ishii, M. Iwakiri, Y. Jang, K. Oshiro in 2013. The axioms of these algebraic systems were motivated by handlebody-knot theory. In the present work we investigate possible constructions which generalise G-family of quandles and other similar constructions (for example, Q- and $(G,*,f)$-families of quandles). We provide the necessary conditions under which the resulting object (called an (α,β,γ)-system) gives a colouring invariant of knotted handlebodies.

We also discuss several other modifications of the proposed construction, providing invariants of spatial graphs with an arbitrary (finite) set of values of vertex valency.

This is joint work with Denis Fedoseev. This research was supported by the Russian Science Foundation under project No. 24-21-00102.

Philipp Korablev

Fibonacci representations of the braid group

In the talk I will describe two representations of the braid group arising from a specific modular fusion category of rank 2. It is planned to describe these representations in a simple combinatorial way without using category theory. It will be shown how the constructed representations can be extended to the invariant of non-oriented links and also to the invariant of closed 3-manifolds.

This research was supported by the Russian Science Foundation under project No. 23-21-10014.

Tatyana Kozlovskaya

Representations of the singular braid groups

We construct a linear representation of singular braid group SB_nwhich is an extension of the Lawrence–Krammer–Bigelow representation and compute the defect of this extension with respect to the exterior product of two extensions of the Burau representation.

The research was supported by the Ministry of Science and Higher Education of Russia (agreement No. 075-02-2024-1437).

Mahender Singh

Separability properties of quandles

In this talk, we will explore residual finiteness and subquandle separability of quandles. The existence of these finiteness properties implies the solvability of the word problem and the generalised word problem for quandles. We will prove that the fundamental n-quandle of any link in the 3-sphere is residually finite for each $n > 1$. This supplements the recent result on residual finiteness of link quandles and the classification of links whose fundamental n-quandles are finite for some n. If time permits, we will state some results on these finiteness properties and give many families of quandles admitting them. This is a recent joint work with Neeraj Dhanwani and Deepanshi Saraf.

Krishnendu Gongopadhyay

Dehn function for the group of palindromic automorphisms

We revisit the group of palindromic automorphisms of the free group. We shall discuss the Dehn function for this subgroup. This is a joint work with Lokenath Kundu.

Nikolai Erokhovets

Manifolds realized as orbit spaces of non-free Z_2^k-actions on real moment-angle manifolds

In dimensions $n = 3$ and 4 we give a criterion when the orbit space of an action of a subgroup H in Z_2^m on the real moment-angle manifold RZ_P coorresponding to a simple n-polytope P with m facets is homeomorphic to a sphere. As a corollary in these dimensions we classify all hyperelliptic invlolutions in Z_2^m/H acting on RZ_P/H generalizing reasults by A.D. Mednykh and A.Yu.Vesnin. In particular, a 3-dimensional small cover has three hyperelliptic involutions if and only if it is a rational homology sphere and if and only it is induced by three hamiltonian cycles such that any edge of P belongs to exactly two of them. For $n = 4$ we classify geometries of the type $R^a \mathrm{x} S^b \mathrm{x} L^c$ realizable on the hyperelliptic

manifolds RZ_P/H. In dimensions $n > 4$ we build examples of spheres and hyperelliptic manifolds of the form RZ_P/H.

The talk is based on the preprint arXiv: 2403.00492.

Igor Nikonov

Biquandloids of knots in thickened surfaces

We define a modification of biquandle construction for knots in a fixed thickened surface and give several examples of the new invariant.

ABSTRACTS OF JUNE 14 TALKS

Andrey Vasiliev

Hypergroups, association schemes and Hall's theorems on finite solvable groups

The concept of a hypergroup, dating back to the work of F. Marti in 1934, generalizes not only the concept of a group, but also an association scheme and table algebra. Within the class of hypergroups, we consider a specific class of hypergroups the members of which we call solvable. Our definition of solvable hypergroups generalizes the notion of a solvable group. We investigate solvable hypergroups and utilize our results to generalize Philip Hall's celebrated theorems on finite solvable groups to the theory of association schemes. The talk is based on the joint paper with P.-H. Zieschang.

Evgeny Fominykh

Minimal ideal triangulations of 3-manifolds

Recent developments in the theory of complexity for 3-manifolds are reviewed. New methods for computing complexity are described, based on calculation of the Turaev-Viro invariants and homologies of 3-manifolds.

This research was supported by the Russian Science Foundation under project No. 22-11-00299.

Dmitri Talalaev

Multivalued quandles: algebra and topology

The report is devoted to a multivalued generalization of the notions of rack and quandle, this generalization largely repeats the multivalued versions of groups and Hopf algebras. Actually, multivalued groups arose from fairly natural problems in topology and were significantly developed in the field of discrete dynamics, algebraic geometry and algebra. In the report, I will talk about the constructions of multivalued quandles, topological problems in which multivalued them occur naturally, and about an associated algebraic object, an analog of the group algebra, for multivalued quandles.

The report is based on the recent joint work with V. Bardakov and T. Kozlovskaya.

Maxim Ivanov

Virtual knot groups and circular orderability

A group G is called left-orderable if there is an order on G that is invariant under left multiplication. There is a related notion of a circularly orderable group, which states whether or not elements of a group G can be "arranged in a circle" in a way that the relative position of the elements of G on a circle is preserved by left multiplication. All classical knot groups are left-orderable by the famous theorem of Howie and Short. Virtual knots were introduced by L. Kauffman as a generalization of classical knots. Virtual knot groups do not have this property in general. We will discuss left-orderability and circular orderability of these groups.

Anton Kazakov

The Kramers–Wannier duality as the Tutte type duality

The Kramers–Vannier duality is a well-known technique that allows us to obtain an equation explicitly determining the value of the critical temperature of the Potts model on the planar square lattice. Also, the Kramers–Vannier duality allows us to obtain an equation that relates the values of critical temperatures of Potts models on a planar lattice Γ and its dual lattice Γ^* with each other [1].

My talk will focus on the derivation of the Kramers–Vannier duality via the Tutte duality [2], which can be applied to Potts models due to their connection with Tutte polynomials [3],[4]. This approach to the Kramers–Vannier duality seems to be very fruitful, since it allows us to consider this duality from a more general algebraic point of view [5], which potentially helps to extend the Kramers–Vannier duality to some Potts models on non-planar lattices.

References

[1] N. Biggs, Interaction models, Vol. 30, Cambridge University Press, (1977).

[2] A.A. Kazakov, Kramers–Vanier duality and Tutta polynomials, approved for printing in TMP.

[3] L. Beaudin, J. Ellis-Monaghan, G. Pangborn, R. Shrock, A little statistical mechanics for the graph theorist. Discrete Mathematics, 310 (13-14) (2010), 2037–2053.

[4] B. Bychkov, A. Kazakov, D. Talalaev, Functional relations on anisotropic Potts models: from Biggs formula to the tetrahedron equation. SIGMA. Symmetry, Integrability and Geometry: Methods and Applications, 17 (2021), 35.

[5] T. Krajewski, I. Moffatt, A. Tanasa, Hopf algebras and Tutte polynomials. Advances in Applied Mathematics, 95 (2018), 271–330.

Andrei Egorov

Upper bounds for the volumes of hyperbolic polyhedra and hyperbolic links

By virtue of Belletti's theorem, the upper exact bound for volumes of generalized hyperbolic polyhedra having the same 1-skeleton G is achieved on an ideal right-angled polyhedron whose 1-skeleton is the medial graph of the graph G. We will talk about the volume estimates for generalized hyperbolic polyhedra that can be obtained using this result. Also we will talk about new upper bounds for the volumes of hyperbolic links that have more than 8 twists in the diagram.

Andrei Malyutin

Quantization effects in groups acting on the circle

We study the group H of self-homeomorphisms of the circle. This group contains the Artin braid groups as subgroups. A recently discovered property of H is the so-called quantization effect. This effect can be described as follows. Say that an element h of H is x-quantizing if h sends the complement of an arc of length x to an arc of length x. Then for arbitrarily small positive x the asymptotically typical element of H is x -quantizing.

Komal Negi

Singular twisted links and singular twisted virtual braids

The concepts of twisted knot theory and singular knot theory inspire the introduction of singular twisted knot theory. This study showcases similar findings for singular twisted links, including the Alexander theorem and the Markov theorem derived from knot theory. Moreover, I will talk about how we define singular twisted virtual braids and their monoid structure. Additionally, we provide both a monoid and a reduced monoid presentation for singular twisted virtual braids.

This is a joint work with my supervisor Dr. Madeti Prabhakar.

Kirandeep Kaur

Virtual knots with unknotting index (n, m)

In [1], K. Kaur et al. posed a problem of finding a virtual knot, if exists, with an unknotting index (n, m), for any pair of non-negative integers (n, m). In this talk, we address this

question by discussing infinite families of virtual knots with unknotting index (n, m), for a given pair of non- negative integers (n, m).

The report is based on the recent joint work with Madeti Prabhakar.

References

[1] K. Kaur, S. Kamada, A. Kawauchi, M. Prabhakar, An unknotting index for virtual knots. Tokyo J. Math., 42(02) (2019), 357–370.

ABSTRACTS OF JUNE 15 TALKS

Mikhail Neshchadim

Constructions of quandles over groups and rings and some questions of group theory

We present the following constructions of extending a trivial quandle by a group with a nontrivial abelian subgroup, a quandle over a noncommutative group by an arbitrary antiautomorphism, a quandle presenting from a generalized Alexander quandle with the replacement of the automorphism by a central antiautomorphism, and a quandle over an n-dimensional module depending on $n(n-1)-1$ parameters with $n \geq 2$ in a commutative unital ring.

Also we study bijections of some groups which are commute with inner automorphisms.

The work was supported by the Fundamental Research Program of the Siberian Branch of the Russian Academy of Sciences (No. I.1.5, Project FWNF2022-0009).

References

[1] A.A. Simonov, M.V. Neshchadim, A.N. Borodin, Constructions of quandles over groups and rings. Siberian Mathematical Journal, 65(3) (2024).

Nikolay Abrosimov

Euclidean volume of a cone manifold over any hyperbolic knot is an algebraic number

We prove that the normalized Euclidean volume of a cone manifold over any hyperbolic knot is an algebraic number, that is a root of a polynomial with integer coefficients. We also provide an algorithm for finding the corresponding minimal polynomial.

Example: Cone manifold over knot 5_2 has normalized Euclidean volume

$$1/\left(6\sqrt{-6+68\sqrt{2}+4\sqrt{983+946\sqrt{2}}}\right) = 0.009909630999945638\ldots$$

Its minimal polynomial is

$$1 + 864\,x^2 - 64457856\,x^4 - 412091172864\,x^6 - 785065068490752\,x^8.$$

The talk is based on our joint work [1] with Alexander Kolpakov and Alexander Mednykh.

The study was carried out within the framework of the state contract of the Sobolev Institute of Mathematics (project no. FWNF-2022-0005).

References

[1] N. Abrosimov, A. Kolpakov, A. Mednykh, Euclidean volumes of hyperbolic knots. Proc. Amer. Math. Soc., 152 (2024), 869–881.

Bao Vuong

On links in Poincare sphere

We study links in Poincare sphere. It is well-known that the Poincare sphere can be obtained by doing surgery on left-handed trefoil knot in 3-sphere S3 with framing -1. Thus, we represent a link in Poincare sphere as a mix link diagram with a surgery component is left-handed trefoil. Further we get a presentation of fundamental group of link complement in Poincare sphere and study classic invariants, related to it such as Alexander matrix, Seifert form, Alexander polynomial.

This is a joint work with Vladimir Evteev.

The research was supported by the Ministry of Science and Higher Education of Russia (agreement No. 075-02-2024-1437).

Matvei Zonov

On verbal solutions of the Yang-Baxter equation

The Yang-Baxter equation (also known as the braid equation) has ties to statistical physics and knot theory. If X is a set and R is a map from X^2 to X^2, then R is said to be a set-theoretic solution of the Yang-Baxter equation if the maps $(\mathrm{R} \times \mathrm{Id})(\mathrm{Id} \times \mathrm{R})(\mathrm{R} \times \mathrm{Id})$ and $(\mathrm{Id} \times \mathrm{R})(\mathrm{R} \times \mathrm{Id})(\mathrm{Id} \times \mathrm{R})$ coincide as maps from X^3 to X^3. In this work, we have investigated verbal maps on two-step nilpotent groups that are solutions of the Yang-Baxter equation.

This research was supported by the Russian Science Foundation under project No. 24-21-00102.

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